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TECHNICAL NOTES

On computing radiative heat flux distributions using the F_N method

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FOR SOLVING radiative transport problems in one-dimensional planar media, the F_N method is an efficient and accurate technique. The approach was developed by Siewert and co-workers [1–3] and has been used for computing heat transfer by a number of investigators [3–14]. So far, however, only boundary heat fluxes have been reported. Results for heat flux distributions throughout the medium have yet to be given. While boundary heat fluxes are readily evaluated in terms of simple weighted sums of the coefficients in the F_N expansion of the exit intensities, heat fluxes internal to the medium must be computed from the primary results associated with the singular eigenfunction expansion. A computational problem is encountered in such evaluations. The purpose of this communication is to demonstrate how this problem may be resolved.

The heat flux within an anisotropically scattering planar slab of finite optical thickness is given from the singular eigenfunction expansion technique by [15]

$$q(\tau) = 2\pi(1-\omega)\left\{\int_{-1}^{1} A(\nu)\nu\, e^{-\tau/\nu}\, d\nu + \sum_{\beta=0}^{k-1} [A(\nu_\beta)\nu_\beta e^{-\tau/\nu_\beta} - A(-\nu_\beta)\nu_\beta\, e^{\tau/\nu_\beta}]\right\}, \quad (1)$$

where the notation of ref. [4] has been used: τ is the optical depth coordinate measured normal to the interface $(0 \leqslant \tau \leqslant \tau_0)$, ω is the scattering albedo, ν_β is a positive, discrete eigenvalue $(\nu_\beta > 1)$, k is the number of such eigenvalues, ν is an eigenvalue in the continuum range $(-1 < \nu < 1)$ and the constants $A(\pm\nu_\beta)$ plus the function $A(\nu)$ are expansion coefficients which are determined from the boundary conditions. The computational difficulty arises in evaluating the integral in the above equation since, as discussed below, it is numerically ill-conditioned for $\nu \to 0$. What is needed is an appropriate representation of the function $A(\nu)$ which avoids numerical singularities in this limit. Such a representation is readily available.

It is to be noted that the function $A(\nu)$ may be evaluated by applying the orthogonality properties of the eigenfunctions at either boundary [4]. In terms of the intensity distribution at the $\tau = 0$ boundary, $I(0, \mu)$, one may write

$$A(\pm\xi) = \frac{1}{N(\pm\xi)}\int_{-1}^{1} I(0,\mu)\phi(\pm\xi,\mu)\mu\, d\mu, \quad 0 \leqslant \xi < 1. \quad (2)$$

However, if only this form for $A(\nu)$ is used, equation (1) will be numerically singular as $\nu \to 0^-$ due to the factor $\exp(-\tau/\nu)$. In terms of the intensity distribution at the $\tau = \tau_0$ boundary one may write

$$A(\pm\xi)\, e^{\mp\tau_0/\xi} = \frac{1}{N(\pm\xi)}\int_{-1}^{1} I(\tau_0,\mu)\phi(\pm\xi,\mu)\mu\, d\mu \equiv \bar{A}(\pm\xi), \quad 0 \leqslant \xi < 1. \quad (3)$$

Again, if only this form for $A(\nu)$ is used, equation (1) will be numerically singular as $\nu \to 0^+$ due to the factor $\exp[(\tau_0-\tau)/\nu]$. To avoid this singular behavior, equation (2) may be used for $\nu > 0$ and equation (3) for $\nu < 0$. When this is done, the integral in equation (1) becomes

$$\int_{-1}^{1} A(\nu)\nu\, e^{-\tau/\nu}\, d\nu = \int_{0}^{1} A(+\xi)\xi\, e^{-\tau/\xi}\, d\xi - \int_{0}^{1} \bar{A}(-\xi)\xi\, e^{-(\tau_0-\tau)/\xi}\, d\xi. \quad (4)$$

In the above equations, μ is the cosine of the altitude angle, ϕ is the eigenfunction for the continuum eigenvalues and N is its norm. These functions are given by (cf. [4])

$$\phi(\nu,\mu) = \tfrac{1}{2}\omega\nu g(\nu,\mu)\mathbb{P}\left(\frac{1}{\nu-\mu}\right) + \lambda(\nu)\delta(\nu-\mu), \quad (5)$$

$$N(\nu) = \nu[\lambda^2(\nu) + \tfrac{1}{4}\pi^2\omega^2\nu^2 g^2(\nu,\nu)], \quad (6)$$

where

$$\lambda(\nu) = 1 + \frac{\omega\nu}{2}\mathbb{P}\int_{-1}^{1}\frac{g(x,x)}{x-\nu}\, dx, \quad (7)$$

$$g(\nu,\mu) = \sum_{l=0}^{L}(2l+1)f_l g_l(\nu)P_l(\mu), \quad (8)$$

in which the symbol $\mathbb{P}$ indicates the Cauchy principal value, the P_l are Legendre polynomials, the $(2l+1)f_l$ are coefficients in the phase function expansion, the $g_l(\nu)$ are simple polynomials in the variable ν and $\delta(x)$ is the delta function.

If the medium is bounded by surfaces which emit diffusely and reflect in a partially specular/partially diffuse manner, the functions $A(+\xi)$ and $\bar{A}(-\xi)$ will be given by (cf. [15])

$$\frac{2N(+\xi)}{\omega\xi}A(+\xi) = \frac{\varepsilon_1\sigma T_1^4}{\pi}B_0(\xi) + \sum_{\alpha=0}^{N} a_\alpha\left[-A_\alpha(\xi) + \rho_1^s B_\alpha(\xi) + 2\rho_1^d\frac{B_0(\xi)}{\alpha+2}\right] \quad (9)$$

$$-\frac{2N(-\xi)}{\omega\xi}\bar{A}(-\xi) = \frac{\varepsilon_2\sigma T_2^4}{\pi}B_0(\xi) + \sum_{\alpha=0}^{N} b_\alpha\left[-A_\alpha(\xi) + \rho_2^s B_\alpha(\xi) + 2\rho_2^d\frac{B_0(\xi)}{\alpha+2}\right] \quad (10)$$

where a_α and b_α are coefficients in the F_N representation of the exit intensities [4, 15], T is temperature, ε is emissivity and ρ^s and ρ^d are specular and diffuse reflectivities. The subscripts 1 and 2 refer to the boundaries at $\tau = 0$ and $\tau = \tau_0$, respectively. The functions $A_\alpha(\xi)$ and $B_\alpha(\xi)$ are defined as

$$A_\alpha(\xi) = \frac{2}{\omega\xi}\int_0^1 \mu^{\alpha+1}\phi(-\xi,\mu)\, d\mu, \quad (11)$$

$$B_\alpha(\xi) = \frac{2}{\omega\xi}\int_0^1 \mu^{\alpha+1}\phi(\xi,\mu)\, d\mu. \quad (12)$$

They are readily computed from recursion relations [4] and are well-behaved for all ξ including $\xi = 0$.

From equations (6)–(10), it is seen that the integrands on the RHS of equation (4) vanish as $\xi \to 0$. Therefore, by using the

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Table 1. Heat flux distributions

	$q(\tau)$					
	Case A		Case B		Case C	
τ	equation (1)	[4]	equation (1)	[13]	equation (1)	[13]
0.0	0.4867	0.4867	0.4167	0.4167	0.90706	0.90706
0.1	0.4120		0.3875		0.8638	
0.2	0.3520		0.3605		0.8250	
0.3	0.3013		0.3351		0.7894	
0.4	0.2574		0.3109		0.7564	
0.5	0.2188		0.2878		0.7257	
0.6	0.1844		0.2655		0.6972	
0.7	0.1531		0.2440		0.6707	
0.8	0.1244		0.2232		0.6460	
0.9	0.0974		0.2030		0.6232	
1.0	0.07145	0.07145	0.1834	0.1834	0.60251	0.60251

Case A: $\omega = 0.2$, $\tau_0 = 1$, $\rho^s = \rho^d = 0.25$, phase function as defined in ref. [4].
Case B: $\omega = 0.8$, $\tau_0 = 1$, $\rho^s = \rho^d = 0.25$, phase function as defined in ref. [4].
Case C: $\omega = 0.8$, $\tau_0 = 1$, $\rho^s = \rho^d = 0$, phase function II defined in ref. [13].

representation given by equation (4), the heat flux defined by equation (1) may be accurately evaluated numerically.

To illustrate the above approach, computations were performed under the same conditions considered in refs. [4, 13]. The reader is referred to these for definitions of the phase functions and the associated discrete eigenvalues. The present results are given in Table 1. They correspond to F_9 computations which converged to five significant digits. The numerical integrations required by equation (4) were performed using Simpson's rule with 200 uniformly equal intervals between 0 and 1. Principal value integrations were achieved by letting a node in the integration scheme coincide with the point of singularity. Then by skipping over that node a numerical approximation to the principal value integral was obtained (since the nodes were equally spaced). Cases A and B in Table 1 give the heat flux distributions corresponding to $\varepsilon_1\sigma T_1^4/\pi = 1$, $\varepsilon_2\sigma T_2^4/\pi = 0$. Each boundary of the slab possesses the same specular and diffuse reflection characteristics ($\rho_1^s = \rho_2^s \equiv \rho^s$, $\rho_1^d = \rho_2^d \equiv \rho^d$). The boundary fluxes derived from ref. [4] are also listed. These are obtained from Tables 3–5 of [4] by noting from equations (33)–(37) of [4] that for boundary emission of this type

$$q(0) = 0.5 + (\rho^s + \rho^d - 1)\theta^-(L) \tag{13}$$

$$q(\tau_0) = (1 - \rho^s - \rho^d)\theta^+(R). \tag{14}$$

Case C in Table 1 gives the heat flux distribution corresponding to a slab having non-reflecting boundaries which is diffusely irradiated at $\tau = 0$ [13] (equivalent to setting $\varepsilon_1\sigma T_1^4/\pi = 1$, $\varepsilon_2\sigma T_2^4/\pi = 0$, $\rho^s = \rho^d = 0$). In all cases it is observed that boundary fluxes computed from equation (4) are in exact agreement with those computed in refs. [4, 13] from the alternate expressions involving the F_N expansion coefficients. Also, the heat flux distributions for cases A and B agree with those computed in ref. [16] from the P_N method. Finally, it is to be noted that the approach developed herein has recently been applied to computing the azimuthally dependent transport problem posed by irradiating a planar medium with a collimated beam at oblique incidence [15, 17].

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Prediction of nucleate pool boiling heat transfer coefficients for binary mixtures

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INTRODUCTION

THE BOILING of binary mixtures is of practical significance for chemical engineering and heat pump applications. So far, no predictive equations of broad generality for the determination of nucleate pool boiling heat transfer coefficients for binary mixtures have appeared in the literature, although numerous experimental investigations were reported. This fact was also mentioned in a recent paper [1]. The object of this study is the derivation of a correlation to determine these heat transfer coefficients. For a detailed literature survey on the subject, the reader is directed to refs. [2, 3]. Only the literature data found to be pertinent to this study will be mentioned.

Sufficient empirical evidence was given in the literature to the effect that, for a given heat flux and pressure, the nucleate pool boiling heat transfer coefficient for a binary mixture can be considerably lower than the molar average of the nucleate pool boiling heat transfer coefficients for the pure components of the mixture. Van Wijk *et al.* [4] gave the following explanation for this heat transfer deterioration: the bubbles leaving the heated surface are enriched in the volatile component (i.e. lower boiling point component). This results in a reduction of this particular component in the boiling boundary layer in the vicinity of the heated surface. The liquid mole fraction of the volatile component in this layer is therefore lower than that in the bulk liquid. Consequently, the boiling temperature in the layer becomes higher than that in the bulk liquid. This can be deduced from a vapour–liquid phase equilibrium diagram of a typical binary mixture. For nucleate pool boiling, the heat transfer coefficient is a function of the wall superheat, i.e. the difference between the wall temperature and the liquid boiling temperature. For the determination of this heat transfer coefficient for a binary mixture, the measured wall superheat (based on bulk liquid boiling temperature) is used, whilst the wall superheat in the boiling boundary layer is driving the flow of heat. The latter is smaller than the former.

The following explanations were also given to clarify the quoted deterioration in heat transfer:

—the change in bubble growth rate caused by the varying resistance to mass transfer of the volatile component in diffusing into a growing bubble [5];
—the increase of wall superheat required to activate bubble nucleation centres for mixtures, resulting in a less-dense bubble population at a given wall superheat as compared with that for pure liquids [6];
—the retardation of two of the three principal heat transport mechanisms (i.e. vapour–liquid exchange and evaporative mechanisms) active in nucleate pool boiling for mixtures [7].

The most popular correlation for the prediction of ΔT_m, the wall superheat at a given heat flux during nucleate pool boiling of a binary mixture, is from Stephan and Körner [8]. This wall superheat is given by the equation:

$$\Delta T_m/\Delta T_i = [1+K\,|y-x|(0.88+0.12\times 10^{-5}P)] \quad (1)$$

where K is an empirical constant different for every binary mixture. ΔT_i in equation (1), the ideal wall superheat, and the

NOMENCLATURE

a	thermal diffusivity of liquid [$m^2\,s^{-1}$]
$b_1,\ldots,b_5$	functions defined in the text
c_p	liquid specific heat [$J\,kg^{-1}\,K^{-1}$]
D	liquid mass diffusivity [$m^2\,s^{-1}$]
h	heat transfer coefficient [$W\,m^{-2}\,K^{-1}$]
K	empirical constant
n	number of data
P	pressure [Pa]
P_c	critical pressure of volatile component [Pa]
q	heat flux [$W\,m^{-2}$]
r	latent heat of evaporation [$J\,kg^{-1}$]
Sn	Scriven number
T	bulk liquid boiling temperature [K]
ΔT	wall superheat, i.e. the difference between wall and bulk liquid boiling temperature [K]
x	mole fraction of liquid for volatile component
y	mole fraction of vapour for volatile component.

Subscripts

1	pure non-volatile component
2	pure volatile component
i	ideal binary mixture
m	real binary mixture
w	evaluation for mass fraction basis.